Threshold Cointegration Tests of the Taylor Rule without the Nuisance Parameters

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Abstract

We develop a new set of threshold cointegration tests such that the asymptotic critical values are either standard normal or χ^2 distributions. As such, there is no need to bootstrap the critical values even in the presence of stationary covariates. We use our tests to examine the validity of the Taylor rule for several different sample periods using real time data. In contrast to the linear model, we find strong evidence of cointegration using a nonlinear Taylor rule with threshold effects. Our estimated threshold models indicate that the Federal Reserve is far more policy active when inflation is high than when it is low. In addition, we find that the response to counteract high inflation was weakest in the 1970s and strongest in the Greenspan era.

Keywords: Taylor Rule, Threshold Cointegration, IV tests

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1. Introduction

A large and growing literature utilizes a threshold regression (TR) to capture the nonlinear relationships present among many macroeconomic variables. As in a linear regression, the estimation results will be spurious unless the nonstationary $I(1)$ variables in the TR are cointegrated. Nevertheless, testing for threshold cointegration is difficult because the appropriate critical values usually depend on a number of nuisance parameters. As evidenced by Li and Madalla (1997), Harris and Judge (1998), Chang, Park and Song (2006), and Swensen (2006), bootstrapping the critical values of a cointegration test can be problematic even in the case of a linear model.

In order to avoid the need to bootstrap a threshold cointegrated system, in Section 2, we develop a new methodology such that inference in a TR can be conducted using a standard normal, or χ^2, distribution. The intuition behind our testing methodology is that we use stationary variables as instruments for the $I(1)$ variables in the threshold regression. As such, the asymptotic distributions of our threshold instrumental variable (IV) cointegration tests, weak-exogeneity tests, and symmetry tests are standard. The distributions remain standard even when stationary covariates are included in the cointegrating relationship. We conclude the section with a number of Monte Carlo experiments showing that our test has reasonable size and power properties.

There are strong reasons to believe that modeling the Taylor rule is especially amenable to our testing methodology. As such, in Section 3, we estimate a Taylor rule for a number of different sample periods using quarterly real time U.S. data. To explain the issues involved, consider the standard linear Taylor rule specification:

$$i_t = r^* + \pi_t + \alpha_1(\pi_t - \pi^*) + \alpha_2 y_t + \alpha_3 i_{t-1} + \alpha_4 i_{t-2} + \varepsilon_t \quad (1)$$

$$= \alpha_0 + \alpha_1\pi_t + \alpha_2 y_t + \alpha_3 i_{t-1} + \alpha_4 i_{t-2} + \varepsilon_t$$

where i_t is the nominal Federal funds interest rate, r^* is the equilibrium real interest rate, π_t is the inflation rate over the last four quarters, π_t* is the central bank's inflation target, y_t is the "output gap" measured as the percentage deviation of real GDP from potential real GDP, $\alpha_0 = r^* - \alpha_1\pi^*$, and ε_t is an *i.i.d.* error term.

The recent macroeconometric literature suggests that a simple OLS or GMM estimation of (1) may not be appropriate. For example, Bunzel and Enders (2007) and Österholm (2005) show that the federal funds rate and the inflation rate act as unit-root processes and that the output gap is stationary. Both papers also employ a battery of Johansen (1988, 1991) cointegration tests and conclude that there is no meaningful linear cointegrating relationship between the inflation rate, output gap, and federal funds interest rate. We reconfirm similar results in this paper using real time data. Moreover, as shown by Bec, Salem, and Collard (2002), Boivin (2006), Taylor and Davradakis (2006), and Qin and Enders (2007), the relationship between the federal funds rate, the output gap, and the inflation rate is likely to be some form of regime-switching model. To the extent that the Federal Reserve is more concerned about high inflation than low inflation, the response of i_t is expected to be more dramatic when the inflation is above the target than when inflation is below the target. Moreover, if it is more difficult for the Fed to reduce inflation than to increase inflation, the response of i_t should be greater for positive values of (π_t - π^*) than for negative values. Since similar arguments can be made for the relationship between i_t and the output gap, it seems reasonable to modify (1) so as to estimate the relationship between i_t, π_t and y_t as a threshold cointegrated system.

The difficulty in using the existing tests to determine whether (1) is a threshold cointegrated relationship is complicated by (a) the presence of the stationary covariate y_t, (b) the presence of the lagged interest rate terms, and (c) the possibility that the variables are jointly endogenous. A researcher might want to incorporate y_t, i_{t-1}, and i_{t-2} into a test for cointegration between i_t and π_t so as to reduce the estimated variance of the error term. Moreover, to our knowledge, the literature does not contain a straightforward system-based threshold cointegration test. However, with our threshold cointegration IV methodology, inference concerning cointegration, weak exogeneity, and threshold behavior is readily conducted without the need to resort to the bootstrap since our test statistics are all standard normal, or χ^2.

To preview the results of Section 3, for each subsample beginning with the Paul Volcker era, our testing procedure indicates the presence of a significant threshold cointegrating relationship. A particularly interesting result is that the Federal Reserve is far more policy active when inflation is high than when inflation is low. While these findings are robust to several different time periods, we find that the Federal Reserve was most aggressive to counteract inflation during the Greenspan era and least aggressive in the 1970s. In Section 4, we summarize our findings and provide concluding remarks.

2. Estimation and Testing Methodology

In this section, we present a general testing methodology for threshold cointegration that avoids any nuisance parameter problems. Consider the following two-regime threshold vector error correction model (ECM):

$$\Delta y_{1t} = [\delta_{11} z_{t-1} + \pi_{11}' q_t + \beta_{11}' s_t] I_{1t} + [\delta_{12} z_{t-1} + \pi_{12}' q_t + \beta_{12}' s_t] I_{2t} + u_{1t} \tag{2a}$$

$$\Delta y_{2t} = [\delta_{21} z_{t-1} + \pi_{21}' q_t + \beta_{21}' s_t] I_{1t} + [\delta_{22} z_{t-1} + \pi_{22}' q_t + \beta_{22}' s_t] I_{2t} + u_{2t} \,. \tag{2b}$$

$$I_{1t} = I(c_t > \tau), \text{ and } I_{2t} = 1 - I_{1t} \,, \tag{2c}$$

where $y_t = (y_{1t}, .., y_{pt})'$ is a p-dimensional $I(1)$ time series (here, $p = 2$), and I_{1t} is the Heaviside indicator such that $I_{1t} = 1$ if $c_t > \tau$ and $I_{1t} = 0$ otherwise; $z_{t-1} = \theta' y_{t-1} = (1, -\beta_0)' y_{t-1} = y_{1,t-1} - \beta_0' y_{2,t-1}$

with the cointegrating vector β_0; and q_t includes a constant term (or all relevant deterministic terms) as well as lagged differenced terms to correct for serial correlation. Note that equations (2a) and (2b) differ from the usual vector threshold ECM, since they include one or more stationary right hand variables, s_t.[1]

The null and alternative hypotheses in the system-based error-correction model (ECM) test for threshold cointegration can be described as follows:

$$H_0\text{: } \delta_{11} = \delta_{12} = \delta_{21} = \delta_{22} = 0, \quad H_1\text{: } H_0 \text{ is not true.} \tag{3}$$

We can apply the usual IV procedure to estimate (2a) and (2b). Specifically, we use the instruments $(w_t I_{1t}, w_t I_{2t})$ for $(z_{t\text{-}1} I_{1t}, z_{t\text{-}1} I_{2t})$, where

$$w_t = z_{t\text{-}1} - z_{t\text{-}m}, \tag{4}$$

and m is a finite number, $m << T$. Clearly, w_t is stationary whether or not the system is cointegrated. Specifically, $w_t = (z_{t\text{-}1} - z_{t\text{-}2}) + (z_{t\text{-}2} - z_{t\text{-}3}) + \ldots + (z_{t\text{-}m+1} - z_{t-m}) = \Delta z_{t\text{-}1} + \Delta z_{t\text{-}2} + \ldots + \Delta z_{t\text{-}m+1}$, where each term in the summation is stationary, even if z_t is $I(1)$ under the null of no cointegration. Notice that w_t acts as an instrument for $z_{t\text{-}1}$ since the two are correlated (under the null and under the alternative) and w_t is uncorrelated with u_{1t} and u_{2t}.

As shown by Banerjee, Dolado and Mestre (1986), it can be advantageous to test for cointegration using an autoregressive distributed lag (ADL) model instead of an ECM. An ADL test is based on the following unrestricted regressions:

$$\Delta y_{1t} = [\alpha_{11} y_{1,t-1} + \beta_{11}' y_{2,t-1} + \pi_{11}' q_t + \beta_{11}' s_t] I_{1t} + [\alpha_{12} y_{1,t-1} + \beta_{12}' y_{2,t-1} + \pi_{12}' q_t + \beta_{12}' s_t] I_{2t} + u_{1t} \tag{5a}$$

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[1] In our analysis of the Taylor rule, we find that the output gap is stationary and we want to incorporate it into the model as a stationary covariate. Note that including stationary covariates in OLS-based cointegration tests is difficult. In OLS-based cointegration tests, the test statistics will critically depend on the nuisance parameter ρ^2 describing the long-run correlation between u_t and v_t, where $v_t = \beta' s_t + u_t$.; see Zivot (2000) and Li (2006) for more detail. This outcome is the same in nature as when adding stationary covariates to unit root tests, as initially suggested by Hansen (1995).

$$\Delta y_{2t} = [\alpha_{21} y_{1,t-1} + \beta_{21}' y_{2,t-1} + \pi_{21}' q_t + \beta_{21}' s_t] I_{1t} + [\alpha_{22} y_{1,t-1} + \beta_{22}' y_{2,t-1} + \pi_{22}' q_t + \beta_{22}' s_t] I_{2t} + u_{2t} \tag{5b}$$

Note that the ADL model does not impose the common factor restriction involved in the ECM model. To see this restriction, note that the first term on the right hand side of the ECM model (2a) is given by $\delta_{11} z_{t-1} = \delta_{11} \theta' y_{t-1} = \delta_{11}(1, -\beta_0)' y_{t-1} = \delta_{11} y_{1,t-1} - \delta_{11} \beta_0' y_{2,t-1}$. This corresponds to imposing the restriction $\delta_{11}\beta_0' = \beta_{11}'$ on the ECM model. The null hypothesis for the system-based ADL test for threshold cointegration is

$$\mathrm{H}_0\text{:}\ \alpha_{11} = \alpha_{12} = \beta_{21} = \beta_{22} = 0. \tag{6}$$

In order to avoid any nuisance parameter problems, our IV ADL tests for threshold cointegration employ the stationary instruments $(w_{1t}I_{1t}, w_{1t}I_{2t}, w_{2t}I_{1t}, w_{2t}I_{2t})$ for the regressors $(y_{1,t-1}I_{1t}, y_{1,t-1}I_{2t}, y_{2,t-1}I_{1t}, y_{2,t-1}I_{2t},)$, where the instruments are

$$w_{1t} = y_{1,t-1} - y_{1,t-m} \text{ and } w_{2t} = y_{2,t-1} - y_{2,t-m}. \tag{7}$$

We also consider a second version of the ADL test based on the null hypothesis first used in Boswijk (1994)

$$\mathrm{H}_0\text{:}\ \alpha_{11} = \alpha_{12} = \beta_{11} = \beta_{12} = 0 \text{ and } \alpha_{21} = \alpha_{22} = \beta_{21} = \beta_{22} = 0. \tag{8}$$

Although our IV methodology works for both the ECM and ADL tests, in the remainder of the paper we focus on the ADL model since the Monte Carlo results reported below indicate that the ADL test has better small sample properties.[2] The joint restrictions defined in (6) and (8) are given by the following expression

$$ADL \text{ or } ADL2 = \Delta y'(\hat{\Sigma} \otimes \tilde{W}(\tilde{W}'\tilde{W})^{-1}\tilde{W}')\,\Delta y\,, \tag{9}$$

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[2] Further details and simulation results for the IV ECM threshold cointegration test are available in the online Appendix at http://ideas.repec.org/s/apl/wpaper.html. The online Appendix also reports the results for a single equation Engle and Granger (1987, EG) IV threshold cointegration test. To conserve space, we do report any of these results here.

where $\Delta y = (\Delta y_1', \Delta y_2')'$ with $\Delta y_1 = (\Delta y_{1m+1},.., \Delta y_{1T})'$ and $\Delta y_2 = (\Delta y_{2m+1},.., \Delta y_{2T})'$. $\widetilde{W}$ is defined by $\widetilde{W} = (\widetilde{W}_{11}', \widetilde{W}_{12}', \widetilde{W}_{21}', \widetilde{W}_{22}')'$, which contains the stacked residuals from the regression of $(w_{1t}I_{1t}, w_{1t}I_{2t}, w_{2t}I_{1t}, w_{2t}I_{2t})'$ on the remaining regressors involving q_t and s_t. The error variance matrix estimate is given by

$$\hat{\Sigma} = T^{-1}\sum \hat{u}_{1t}'\hat{u}_{2t} = \hat{\Sigma}^{1/2}\hat{\Sigma}^{1/2}', \tag{10}$$

where $\hat{\Sigma}^{1/2}$ is the Cholesky decomposition of a consistent estimate of the error variance $\hat{\Sigma}$, and $\hat{u}_{1t}$ and $\hat{u}_{2t}$ are the residuals from regressions (5a) and (5b). Although the residuals are obtained from IV regressions, the estimates of the slope coefficients do not require the use of this error variance since the same regressors are used in both systems. However, the relevant Wald statistics should incorporate information of the correlation structure of the error variance. Note that the Wald statistic in (9) is equivalent to the Wald statistic derived from the usual 3SLS estimation using the relevant instruments in each equation of the system, and has the standard chi-square asymptotic distribution.

Theorem 1. Suppose that the data generating process implies (5a) and (5b) with $\alpha_{11} = \alpha_{12} = \alpha_{21} = \alpha_{22} = 0$. Then, under the null hypothesis, each test has the standard chi-square distribution

$$ADL \rightarrow \chi^2_4, \text{ and } ADL2 \rightarrow \chi^2_8 .$$

The proof is in the Appendix.

Most important, the distribution is free of any nuisance parameters. Note also that the distribution does not depend on the number of regressors, the threshold indicator function, and the threshold parameters. The threshold variable can be any $I(1)$ or $I(0)$ variable, or combination of variables. Whereas previous papers on threshold cointegration rely on the error correction term z_{t-1}, or its change Δz_{t-1}, as the threshold variable, we can model the threshold variable in a more flexible form. For example, when we estimate the Taylor rule, we let the threshold depend on a weighted average of the past inflation rate and the output gap. Throughout, we assume that the threshold parameter τ is known *a priori*, or we estimate τ from the data prior to performing our tests.[3]

We also consider corresponding single equation IV threshold cointegration tests. If the variable y_{2t} is weakly exogenous, a single equation test can be especially powerful since it allows the researcher to add Δy_{2t} as a regressor in the equation for Δy_{1t}:

$$\Delta y_{1t} = [\alpha_{11} y_{1,t-1} + \beta_{11}' y_{2,t-1} + \pi_{11}' q_t + d_{11} \Delta y_{2t} + \beta_{11}' s_t] I_{1t} + \tag{11}$$

$$[\alpha_{21} y_{1,t-1} + \beta_{12}' y_{2,t-1} + \pi_{12}' q_t + d_{12} \Delta y_{2t} + \beta_{12}' s_t] I_{2t} + u_t$$

As in the system based tests, we consider two different null hypotheses

H_0: $\alpha_{11} = \alpha_{12} = 0$,

and

H_0: $\alpha_{11} = \alpha_{12} = 0$ and $\beta_{11} = \beta_{12} = 0$.

We denote the resulting tests as ADL^* and $ADL2^*$, respectively, where "*" denotes the single equation test. The single equation tests are described by the following expression

$$ADL^* \text{ or } ADL2^* = \Delta y_1' \tilde{W}' [\hat{\sigma}_1^2 (\tilde{W}' \tilde{W})^{-1}] \tilde{W}' \Delta y_1 \, , \tag{12}$$

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[3] We do not consider an endogenous type threshold cointegration test where supreme tests or other form of order statistics are employed to identify the threshold parameter. In some cases the threshold parameter may be known *a priori*. If not, we can use the usual procedures such as minimizing the sum of squared residuals or information criteria to determine the optimal value of the threshold parameter and take this value as given prior to performing our tests.-The IV test statistics that we propose do not depend on the threshold (percentile) parameter, as they have standard normal or chi-square distributions throughout. However, if we adopt supreme or other order statistics we will lose this invariance property and size distortions will result. Similar perils can arise in endogenous unit root tests; see, for example, Lee and Strazicich (2003). Fortunately, the estimated threshold parameter is usually approximately the median value or in the middle inter-quartile range of possible threshold values. This value can be adopted as the threshold parameter and taken as given.

where $\widetilde{W}$ is defined as in the system based tests in (12). $\widetilde{W}$ are the residuals from the regression of the instrumented variables on the other regressors including q_t, $\Delta y_{2t,}$ and s_t, and $\hat{\sigma}_1^2$ is the estimated error variance of u_{1t} in (11).

Corollary 1. Suppose that the data generating process implies (11) with $\alpha_{11} = \alpha_{21} = 0$. Then, under the null hypothesis

$$ADL^* \rightarrow \chi_2^2 \text{ , and } ADL2 \rightarrow \chi_4^2 .$$

The proof is in the Appendix.

One may also examine the usual IV t-statistics on the coefficients of $z_{t\text{-}1}$, $y_{1,t\text{-}1}$, and $y_{2,t\text{-}1}$ in any system or single equation tests using (5a), (5b), and (11), since the asymptotic distribution is standard normal.

Corollary 2. $t_{iv} \rightarrow N(0, 1)$.

The proof is in the Appendix.

Given the above result, we can also examine the usual t-statistic on $\alpha_{ij} = 0$, or $\beta_{ij} = 0$ to test the significance of the parameter estimates in each regime. Similarly, we can test for non-linearity by using a t-test in (5a), (5b), and (11). For instance, for the regression in (5a), one may test the hypothesis

H_0: $\alpha_{11} = \alpha_{12}$ (linear) vs. H_a: $\alpha_{11} \neq \delta\alpha_{12}$ (nonlinear).

The asymptotic distribution of this t-statistic is standard normal.

Finite Sample Performance

In order to examine the small-sample properties of our test, we perform several Monte Carlo experiments using the data generating process (DGP) from Kremers *et al*. (1992):

$$\Delta y_{1t} = \varphi'\Delta y_{2t} + \delta\,(y_{1t\text{-}1} - d_{t\text{-}1} - \theta' y_{2t\text{-}1}) + \beta s_t + v_t, \qquad (13)$$

$\Delta y_{2t} = u_t$,

$s_t = \rho s_{t\text{-}1} + \varepsilon_t$,

where $v_t \sim N(0, \sigma_v^2)$, $u_\mathrm{t} \sim N(0, \sigma_u^2)$, and $\varepsilon_t \sim N(0, \sigma_\varepsilon^2)$. Although we assume that v_t and u_t are uncorrelated, this assumption is not important since our system based tests since these tests explicitly control for contemporaneous correlations between the error terms. Notice that ρ captures the persistence of the stationary covariates and that δ indicates the strength of the cointegrating relationship. Under the null, $\delta = 0$ and under the alternative, $\delta < 0$. In order to conserve space, we report simulation results for 5000 Monte Carlo replications using $\rho = 0.9$, $T = 100$, and $\delta = [\ 0, -0.1\ ]$. Moreover, we report the size and power of our tests using the asymptotic chi-square distribution at the 5% significance level. The value of m is selected to minimize the sum of squared residuals.

We consider four indicators with different threshold variables: $z_{t\text{-}1}$, $\Delta z_{t\text{-}1}$, $I(1)$ and $I(0)$. $z_{t\text{-}1}$ and $\Delta z_{t\text{-}1}$ models the usual asymmetric level and momentum threshold effects, respectively, while $I(1)$ and $I(0)$ models any other $I(1)$ or $I(0)$ threshold variable from inside or outside the model. We examine the effect of using different signal-noise ratios on the size and power of each test, where $\sigma_\mathrm{v}^2 = 1$ and $\theta = 1$, and denote $sig = \sigma_\mathrm{u}/\sigma_\mathrm{v}$ with the signal-noise ratio $r = -(\varphi-1)\mathrm{sig}$. We examine cases with $(\varphi, \mathrm{sig}) = (1.0, 1)$, $(0.5, 6)$, and $(0.5, 16)$, which corresponds to $r = 0$, 3, and 8, respectively. We first examine the single equation tests, denoted as ADL^*, $ADL2^*$ without stationary covariates. These simulation results are displayed in the left side of Table 1. It is clear that all IV threshold cointegration tests are invariant to the signal-noise ratio under the null $\delta = 0$. All tests show a mild downward size distortion (under rejection), except when the threshold variable is $I(1)$. Overall, the tests have reasonable size properties with virtually no over rejections. It is important to note that the tests have impressive power so long as the signal/noise ratio differs from zero and that the power increases dramatically as the signal-noise ratio increases. The intuition is clear: the contemporaneous regressor Δy_{2t} in equation (13) acts as a stationary covariate and the power of the tests increases as the signal-noise ratio (r) increases. This phenomenon can be explained in the same spirit as when adding stationary covariates to unit root tests as described by Hansen (1995). However, the OLS based unit root tests with stationary covariates have a nuisance parameter problem and OLS based cointegration tests will have a similar problem. Although it is usually difficult to find desirable stationary covariates in unit root tests, this is not the case here, since Δy_{2t} is included in the IV testing regression. While size and power properties differ somewhat depending on the particular threshold variable, the differences are small. We next examine the system based IV threshold cointegration tests, which are denoted as *ADL* and *ADL*, respectively. While the DGP still follows equation (13), we exclude Δy_{2t} from the system based testing regression. While the assumption of weak exogeneity is implied in equation (13), the results are hardly affected by relaxing this assumption in the system based tests. The results without stationary covariates are displayed in the right hand side of Table 1 and are very similar to those for the single equation tests. Under the null, we again observe mild negative size distortions (under rejections) and no over rejections. However, the system based tests are more powerful than the single equation tests in nearly all cases.

In Table 2, we examine the effect of adding the stationary covariate variable s_t to the testing regression. The results under the null are not much different from those in Table 1. We still observe mild negative size distortions (under rejections) and sizes that are unaffected by the signal-noise ratio. In addition, the sizes do not depend on the values of the coefficients ρ and β in (18) that describe the stationary covariates. Most important, the power is larger when the stationary covariate s_t is included, and increases monotonically as the value of β increases and/or as the signal-noise ratio increases. The size properties are virtually unchanged regardless of the threshold variable that is adopted. We next examine the results in Table 3 for the system based tests with the stationary covariate included. Again, the results are very similar to those in the single equation tests. When stationary covariates are included, the power mostly increases, despite the under-rejections under the null. Again, the size and power properties show only small differences for different indicator variables.

3. Empirical Results

In order to estimate a Taylor rule in the form of (1), we obtained monthly observations of the federal funds rate from the Federal Reserve Bank of St. Louis' data base FRED II (http://research.stlouisfed.org/fred2/). Unlike interest rates, price and output data can be subject to substantial revisions. As pointed out by Orphanides (2001), to best ascertain the behavior of the Federal Reserve, it is necessary to use the data actually available to the Fed at the time their decisions are made. As such, we obtained real-time data on real output and the output price index, p_t, from the Philadelphia Federal Reserve Bank's website (http://www.phil.frb.org/econ/forecast/reaindex.html).

Since the output and price data is reported quarterly and the interest rate data is reported monthly, we use the quarterly average of the federal funds rate as the dependent variable i_t. We follow standard practice and construct an inflation measure, π_t, as the average inflation rate over the past year. Specifically,

$$\pi_t = 0.25\sum_{i=0}^{3}\pi_{t-i}^{q} \quad \text{where } \pi_t^q = 400(\ln p_t - \ln p_{t-1})$$

and, for each π_t, all values of p_{t-i} are the real-time price indices for time period t-i reported at time period t.

Several different methods have been proposed to measure the output gap, y_t. Since we use the data set originally constructed by Croushore and Stark (1999), we adopt their methodology and filter the real output data with an HP filter. As indicated in Croushore and Stark (1999), our aim is not to ascertain the way that real output evolves over the long-run. Instead, the goal is to obtain a reasonable measure of the pressure felt by the Federal Reserve to use monetary policy to affect the level of output. Since we use real time data, we first HP filter the entire real-time output series available for each time period t. We construct y_t as the percentage difference between the values of real-time output and the HP filtered output.[4]

Results Using Linear Models

We first examine the Taylor rule in the linear framework. In Table 4, we report estimates of the Taylor rule for five sample periods often examined in the literature. The 1970:1 – 1979:2 period represents the tenure of William Martin, Arthur Burns and G. William Miller as Federal Reserve chairmen. The so-called Volcker-Greenspan period spanned 1979:4 – 2005:4. A number of empirical papers omit the first few years of this period since the Fed experimented with money base targeting in the early part of the Volcker period. The pure Greenspan period begins in 1987:4 and ends in 2005:4.

There are several important issues to note about the estimates shown in Table 4. For all time periods, the estimated coefficients on π_t and y_t are positive and usually significant at conventional levels. However, there are some problems with the estimates that might be dubbed "Some unpleasant Taylor rule arithmetic." Specifically:

- The coefficients, or the sum of the coefficients, on the lagged federal funds rate are all quite large. Although this is often referred to as 'interest rate smoothing,' the amount of

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[4] Hence, a positive value of y_t means that real-time output exceeds the level of 'potential' output.

smoothing seems excessive. In fact, the estimates are close enough to unity to suggest the possibility of a unit-root in the federal funds rate. Of course, it could also be argued that i_t, π_t and y_t are jointly endogenous variables so that GMM estimation is preferable to OLS estimation. Nevertheless, there are several other problems that are unlikely to be resolved by GMM estimation.

- As shown in Table 5, diagnostic checking indicates that the interest rate and inflation variables act as unit root processes. Specifically, Dickey-Fuller tests indicate that for all sample periods ending in 2005:4 it is not possible to reject the null hypothesis of a unit root plus drift (against an alternative of stationary about a fixed mean) in either i_t or π_t. When intercepts are included in the unit root test equations, the output gap appears to be stationary over the entire sample period, but it is not clear whether y_t acts as a stationary process during the 1970:1 – 1979:2 and 1987:4 – 2005:4 sub-periods. A well-known problem of the Dickey-Fuller test is that it loses power in the presence of deterministic regressors in the estimating equation that are not in the data generating process. For all sample periods under consideration, the HP filtered output gap had a mean near zero. Moreover, the intercepts in the Dickey-Fuller τ_μ test were all insignificant at any conventional significance level. When these insignificant intercept terms were eliminated from the τ_μ version of the Dickey-Fuller test, the null hypothesis of a unit root in y_t could be rejected at the 1% level in all sub-periods. Therefore, we will proceed with the assumption that y_t is stationary in all periods.
- The possibility of nonstationary variables that the Taylor rule equation is spurious unless the $I(1)$ variables are cointegrated. Even if the variables turn out to be cointegrated, the distribution of the coefficient estimates obtained by using OLS (or GMM) do not have the usual properties since they multiply nonstationary variables.

Given the above, the first important task is to check for the existence of a cointegrating relationship among the nonstationary variables. Respectively, Tables 6 and 7 report the results of the Engle-Granger and Johansen linear cointegration tests for the various sample periods.[5] At conventional significance levels, the null hypothesis of no cointegration could not be rejected in any of the sample periods. When we used the Johansen test, the null hypothesis of no cointegration could be rejected for the Volcker-Greenspan period only (1979:4 – 2005:4). For this period, the sample values of λ_{max} and λ_{trace} of 18.50 and 22.13 exceeded the 5% asymptotic critical values of 15.67 and 19.96. Nevertheless, the estimated cointegrating relationship is problematic since the long-run relationship is $i_t = -59.666 + 29.092\pi_t$ and the speed of adjustment coefficients are both positive. The only evidence of a reasonable cointegrating relationship was for the Greenspan period (1987:4 – 2005:4) when we checked for cointegration between i_t, π_t and y_t. When y_t was treated as $I(1)$, the estimated cointegrating relationship for this period is:

$$i_t = -5.783 + 4.283\pi_t + 6.991\, y_t\, .$$

With three variables in the potential cointegrating relationship, the sample values of λ_{max} and λ_{trace} of 24.63 and 36.69 exceeded the 5% asymptotic critical values of 22.00 and 34.91, respectively. There was no strong evidence of a second cointegrating vector. Inflation and the output gap appear to be weakly exogenous, while the speed of adjustment coefficient for i_t was –0.075 with a t-statistic of –4.965. Yet, some would argue with this finding since it is unlikely that y_t actually contains a unit root. Still more would be concerned that the sample period contains only 65 observations whereas each equation in the 8-lag VAR has 40 coefficients.

Threshold Estimation

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[5] In the tables, we exclude the 1970:1 – 1979:2 sample period since i_t was found to be stationary over this period.

We began by estimating a baseline threshold model using Chan's (1993) method to obtain a consistent estimate of the threshold variable τ. The form of the baseline model is

$$i_t = [\alpha_0 + \alpha_1\pi_t + \alpha_2 y_t + \alpha_3 i_{t-1} + \alpha_4 i_{t-2}]I_{1t} + [\beta_0 + \beta_1\pi_t + \beta_2 y_t + \beta_3 i_{t-1} + \beta_4 i_{t-2}]I_{2t} + \varepsilon_t\ .$$

where I_{1t} is the Heaviside indicator that $I_{1t} = 1$ if $\pi_{t-d} > \tau$ and $I_{1t} = 0$ otherwise, $I_{2t} = 1 - I_{1t}$; and d is the delay parameter estimated as the integer value of $d = 1$ and 2 that results in the smallest residual sum of squares.

The results for all five sample periods are shown in Table 8. Consider the estimates for the 1979:4 – 2005:4 sample period

$$i_t = [\underset{(3.22)}{6.023} + \underset{(3.92)}{0.588}\,\pi_t + \underset{(5.27)}{1.356}\,y_t + \underset{(5.31)}{0.458}\,i_{t-1}]I_{1t} + [\underset{(-4.20)}{-0.088} + \underset{(1.69)}{0.196}\,\pi_t + \underset{(3.61)}{0.232}\,y_t + \underset{(19.59)}{0.977}\,i_{t-1}]I_{2t}$$

where $d = 1$ and $\tau = 5.517$.

In the threshold model, both the intercept terms and the slope coefficients determine the degree of feedback between π_t and i_t. For any given value of i_{t-1}, as π_{t-1} begins to exceed the threshold of value 5.517, i_t increases by (6.023 – 0.088) + (0.588 – 0.196)π_t + (1.336 – 0.232) y_t. For all values of π_t and y_t in the data set, the increase in i_t is sufficient to ensure that $i_t - \pi_t$ rate is much greater when $\pi_t \geq 5.517$ than when $\pi_t < 5.517$. Also notice that there is far more interest rate smoothing when inflation is below the threshold than when it is above the threshold. Overall, the point estimates of the coefficients suggest that the Fed is far more policy active in the high inflation regime than in the low inflation regime.

Although these results are interesting, the properties of the parameter estimates are unknown because the estimated coefficients do not have a standard normal distribution and it is not clear whether the regression equations are spurious. Finally, the estimates of the Taylor rule might be problematic because of the possibility that i_t, π_t and y_t are jointly endogenous. In order to alleviate some of these problems, we implement our IV threshold cointegration tests to estimate the coefficients and test the validity of the model.

IV System Estimates

We begin by estimating i_t and π_t as a bivariate threshold ADL system of equations in the form

$$\Delta i_t = [\alpha_{10} + \alpha_{11} i_{t\text{-}1} + \alpha_{12}\pi_{t\text{-}1} + \alpha_{13} y_{t\text{-}1}]I_{1t} + [\beta_{10} + \beta_{11} i_{t\text{-}1} + \beta_{12}\pi_{t\text{-}1} + \beta_{13} y_{t\text{-}1}\ ]I_{2t} + \varepsilon_{1t}$$

$$\Delta \pi_t = [\alpha_{20} + \alpha_{21} i_{t\text{-}1} + \alpha_{22}\pi_{t\text{-}1} + \alpha_{23} y_{t\text{-}1}]I_{1t} + [\beta_{20} + \beta_{21} i_{t\text{-}1} + \beta_{22}\pi_{t\text{-}1} + \beta_{23} y_{t\text{-}1}\ ]I_{2t} + \varepsilon_{2t}\ ,$$

where $I_{1t} = 1$ if $\pi_{t\text{-}d} > \tau$, $I_{1t} = 0$ if $\pi_{t\text{-}d} \leq \tau$, and $I_{2t} = 1 - I_{1t.}$

Notice that we use the same threshold value τ for each of the two equations. In order to test for cointegration we use the instruments $I_{1t}(i_{t\text{-}1} - i_{t\text{-}m})$, $I_{2t}(i_{t\text{-}1} - i_{t\text{-}m})$, $I_{1t}(\pi_{t\text{-}1} - \pi_{t\text{-}m})$, and $I_{2t}(\pi_{t\text{-}1} - \pi_{t\text{-}m})$ for the variables $I_{1t}i_{t\text{-}1}$, $I_{2t}i_{t\text{-}1}$, $I_{1t}\pi_{t\text{-}1}$, and $I_{2t}\pi_{t\text{-}1}$.[6] As such, the distributions of α_{11}, β_{11}, α_{12}, β_{12}, α_{21}, β_{21}, α_{22}, and β_{12}, are multivariate normal. This is especially useful since we can test for cointegration in a straightforward fashion. If we can reject the restriction that $\alpha_{11} = \beta_{11} = 0$, it follows that the interest rate and inflation rate are cointegrated. If, at the same time, we cannot reject the restriction $\alpha_{12} = \beta_{12} = 0$, then the suggestion is that the inflation rate is weakly exogenous. If all eight values of $\alpha_{ij} = \beta_{ij} = 0$ for $i = 1, 2$ and $j = 1, 2$, we can conclude that the interest rate and inflation rate are not cointegrated. In the form presented above, the residuals of the two ADL equations had serial correlation; we corrected this problem by using lagged values

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[6] For each time period, we use the values of d shown in Table 1. We selected m as follows. For each period, we obtained the estimated threshold values shown in Table 4. Then, holding the threshold value constant, we performed the IV estimation using values of $m = 4, \ldots, 10$. We then selected the value of m resulting in the smallest residual variance. Using this value of m, we re-estimated the threshold value τ.

of the dependent variables.[7] The estimates for the four sample periods exhibiting threshold behavior are shown in columns 2 through 5 of Table 9. Notice:

- The system-based estimates of the Taylor rule perform poorly over the 1983:1 – 2005:4 period. The sample values of the *F*-statistics for the null hypotheses $\alpha_{11} = \beta_{11} = \alpha_{12} = \beta_{12} = 0$, $\alpha_{11} = \beta_{11} = 0$, and $\alpha_{12} = \beta_{12} = 0$ have *prob*-values of 0.839, 0.774 and 0.793, respectively. As such, it seems that the Taylor rule is invalid for this period.
- The null hypothesis of no threshold cointegration is strongly rejected for the 1979:4 – 2005:4 and 1987:4 – 2005:4 periods. For the 1970:1 – 2005:4 period, the *F*-statistic for the null hypothesis $\alpha_{11} = \beta_{11} = 0$ has a *prob*-value of 0.034, although the *F*-statistic for the null hypothesis $\alpha_{11} = \beta_{11} = \alpha_{12} = \beta_{12} = 0$ has a *prob*-value of 0.095.
- For the Volcker-Greenspan and the Greenspan periods, the response of Δi_t to inflation is greater when the rate of inflation is above the threshold rather than below. For example, during the Greenspan period, the coefficient on $I_{1t}\pi_{t-1}$ is estimated to be 1.239 whereas the coefficient on $I_{2t}\pi_{t-1}$ is estimated to be 0.023. Also notice that there is far more interest rate smoothing when inflation is below the threshold than when it is above the threshold. For example, in the Greenspan period, the coefficient on $I_1\pi_{t-1}$ is –0.618 and the coefficient on $I_2\pi_{t-1}$ is –0.098. Hence, Δi_t responds to its lagged level far more in the high inflation regime than in the low inflation regime.
- Except for the Greenspan period, the coefficients on the output gap in the Δi_t equations are generally of the same magnitude and the *t*-tests suggest that they are insignificant. However, during the Greenspan period, the response of Δi_t to $I_{1t}y_{t-1}$ was 0.921 and the

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[7] For brevity, we do not report the stationary dynamics in the tables or in the text. Since degrees of freedom quickly become a problem in a threshold system of equations, we used only one or two lags of Δi_{t-i} in the first equation and only one or two lags of $\Delta \pi_{t-i}$ in the second equation.

response to $I_{2t}y_{t-1}$ was only 0.245. Thus, in high inflation periods, the effects of a positive output gap had a magnified effect on the behavior of the federal funds rate during the Greenspan era.

- Except for the Greenspan period, there is little evidence that $\Delta\pi_t$ adjusts to an equilibrium level since the null hypothesis $\alpha_{22} = \beta_{22} = 0$ cannot be rejected at conventional significance levels. For the Greenspan period, the null hypothesis that $\alpha_{22} = \beta_{22} = 0$ has a *prob*-value of 0.004.

Single Equation Estimates

Since the values of $I_{1t}i_{t-1}$ and $I_{2t}i_{t-1}$ can be excluded from the $\Delta\pi_t$ equation for the 1979:4 – 2005:4 period, it seems unnecessary to estimate the equations as a system. Instead, we can estimate Δi_t as a single equation ADL test using the same IV variables as before. The advantage of this method is that the weak exogeneity of the inflation rate means that $I_{1t}\Delta\pi_t$ and $I_{2t}\Delta\pi_t$ can be included as additional stationary covariates in the testing regression. Consider the estimated single equation ADL threshold cointegration test of the nonlinear Taylor Rule for the 1979:4 – 2005:4 time period:

$$\begin{aligned}\Delta i_t = {} & [\underset{(1.24)}{2.247} - \underset{(-3.18)}{0.718}\, i_{t-1} + \underset{(3.43)}{1.032}\,\pi_{t-1} + \underset{(2.76)}{0.383}\, y_{t-1} + \underset{(0.59)}{0.349}\,\Delta\pi_t] I_{1t} \\ & + [\underset{(0.17)}{0.123} - \underset{(-1.64)}{0.062}\, i_{t-1} + \underset{(0.17)}{0.047}\,\pi_{t-1} + \underset{(6.37)}{0.289}\, y_{t-1} + \underset{(1.85)}{0.378}\,\Delta\pi_t] I_{2t}\end{aligned}$$

The coefficients of $I_{1t}i_{t-1}$ and $I_{2t}i_{t-1}$ are both negative and the *t*-statistics indicate that both are significantly different from zero using a one-sided test. As shown in Table 10, the test for the joint restriction that both coefficients equal zero has a *prob*-value of 0.002 and the test for $\alpha_{11}=\beta_{11}=\alpha_{12}=\beta_{12}=0$ also has a *prob*-value of 0.001. As such there is strong evidence of a cointegrating relationship with the inflation rate acting as a weakly exogenous variable. Unlike with the system equation estimates, there is now strong evidence that the federal funds rate responds to the output gap. Notice that the response is quite similar across the two regimes. For completeness, in Table 10 we also report the single-equation IV estimates for the other sample periods (with and without $I_{1t}\Delta\pi_t$ and $I_{2t}\Delta\pi_t$) even though the inflation rates may not be weakly exogenous in these other periods.

Other Threshold Functions

We were earlier concerned that the Taylor rule estimates are spurious for the 1983:1 – 2005:4 period even though the rule seems to work for the surrounding 1979:4 – 2005:4 and 1987:4 – 2005:4 periods. At the risk of over-fitting the data, we experimented with several other possible threshold functions. Overall, no single threshold variable worked as well as the inflation rate. However, a simple modification of Taylor's original specification, such that the threshold is a weighted average of π_{t-d} and y_{t-d}, yielded interesting results for the 1983:1 – 2005:4 period. Consider the following threshold function

$I_t = 1$ if $w\pi_{t-1} + (1 - w)y_{t-1} > \tau$ and $I_t = 0$ otherwise; $0 \leq w \leq 1$.

The idea is that the regime change depends on a threshold variable that is a combination of the inflation rate and output gap. Taylor's original specification suggested a weight of 0.5. However, experimentation with values of w equal to 0, 0.25. 0.5, and 0.75 indicated that the value $w = 0.75$ resulted in the best fit (as measured by the log of the determinant of the variance covariance matrix). The results for the 1983:1 – 2005:4 period are shown in the last column of Table 9. Notice that the point estimates of α_{11} and β_{11} are both negative and the joint hypothesis $\alpha_{11} = \beta_{11} = 0$ is clearly rejected at conventional significance levels. As such, we conclude that the Federal Reserve also utilized the output gap as a threshold variable during at least part of the 1983:1 – 2005:4 period. The coefficient on $I_{1t}\pi_{t-1}$ is 0.676 and has a *t*-statistic of 2.243. The joint hypothesis that $\alpha_{12} = \beta_{12} = 0$ has a *prob*-value of 0.077. The point estimates of α_{13} and β_{13} are individually significant. Notice that the full response of the Fed's reaction to changes in y_{t-1} depend on the slope coefficients for $I_{1t}y_{t-1}$ and $I_{2t}y_{t-1}$ *and* on the intercepts since changes in y_{t-1} can induce the system to cross the threshold value of 2.72. Also notice that we cannot reject the null hypothesis that $\Delta\pi_t$ is error-correcting since the joint hypothesis $\alpha_{22} = \beta_{22} = 0$ has a *prob*-value of 0.617.

4. Conclusion

We seek to fill an important gap in the literature by developing new threshold cointegration tests using stationary instrumental variables. We consider both single equation and system based tests and use these tests to estimate and test the validity of a nonlinear Taylor rule. An important advantage of our testing procedure is that the test statistics are asymptotically standard (normal or chi-square) in all cases and are free of the nuisance parameter problems found in other tests. Monte Carlo experiments indicated that the tests have reasonably good size and that the power can be substantial when stationary covariates are included in the model.

We applied our testing methodology to several versions of the U.S. Taylor rule using quarterly real time data from 1970-2005. In our nonlinear models, the behavior of the monetary authority is hypothesized to depend on whether inflation is higher or lower than a threshold rate. Our test results find significant evidence of cointegration in the nonlinear models. The estimated coefficients indicate that the Federal Reserve will increase the federal funds interest rate strongly when inflation exceeds a threshold. However, when inflation is below the threshold, there is substantial policy inertia. We also find that the Federal Reserve was least aggressive to counteract inflation during the 1970s and most aggressive in the Greenspan era. We additionally find that a threshold variable combining inflation and the output gap can better explain monetary policy during the Greenspan era.

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Table 1: Size and Power of Single Equation and System Based IV Threshold Cointegration Tests
(Without Stationary Covariates)

δ	I_t	r	ADL^*	$ADL2^*$	ADL	$ADL2$
0.0	ΔZ_{t-1}	0	0.024	0.018	0.018	0.011
		3	0.024	0.018		
		8	0.024	0.018		
	Z_{t-1}	0	0.012	0.008	0.013	0.008
		3	0.012	0.008		
		8	0.012	0.008		
	I(0)	0	0.025	0.019	0.022	0.012
		3	0.025	0.019		
		8	0.025	0.019		
	I(1)	0	0.051	0.032	0.038	0.027
		3	0.051	0.032		
		8	0.051	0.032		
-0.1	ΔZ_{t-1}	0	0.113	0.109	0.295	0.277
		3	0.625	0.890	0.870	0.993
		8	0.947	1.000	0.992	1.000
	Z_{t-1}	0	0.031	0.014	0.085	0.077
		3	0.290	0.675	0.680	0.978
		8	0.892	0.998	0.993	1.000
	I(0)	0	0.117	0.066	0.321	0.294
		3	0.653	0.899	0.885	0.995
		8	0.951	1.000	0.989	1.000
	I(1)	0	0.173	0.096	0.322	0.296
		3	0.612	0.884	0.855	0.996
		8	0.945	1.000	0.855	0.996

Notes: "*" denotes single equation tests. Details of the simulations are described in Section 2.

Table 2: Size and Power of Single Equation Based IV Threshold Cointegration Tests (With Stationary Covariates)

δ	I_t	r	ρ	β	ADL^*	$ADL2^*$	δ	ADL^*	$ADL2^*$
Size							Power		
0.0	ΔZ_{t-1}	0	0.9	1	0.012	0.011	-0.1	0.657	0.773
		3			0.012	0.011		0.768	0.900
		8			0.012	0.011		0.939	0.998
		0	0.9	3	0.014	0.010		0.967	0.991
		3			0.014	0.010		0.972	0.993
		8			0.014	0.010		0.979	0.997
	Z_{t-1}	0	0.9	1	0.012	0.011		0.558	0.604
		3			0.012	0.011		0.641	0.787
		8			0.012	0.011		0.907	0.995
		0	0.9	3	0.013	0.009		0.965	0.990
		3			0.013	0.009		0.967	0.991
		8			0.013	0.009		0.976	0.998
	I(0)	0	0.9	1	0.014	0.014		0.670	0.741
		3			0.014	0.014		0.788	0.907
		8			0.014	0.014		0.945	0.999
		0	0.9	3	0.014	0.012		0.972	0.993
		3			0.014	0.012		0.974	0.993
		8			0.014	0.012		0.984	0.999
	I(1)	0	0.9	1	0.015	0.013		0.462	0.384
		3			0.015	0.013		0.755	0.892
		8			0.015	0.013		0.954	0.999
		0	0.9	3	0.013	0.011		0.940	0.955
		3			0.013	0.011		0.953	0.976
		8			0.013	0.011		0.978	0.998

Notes: Details of the simulations are described in Section 2.

Table 3: Size and Power of System Based IV Threshold Cointegration Tests
(With Stationary Covariates)

I_t	r	ρ	β	Size ($\delta = 0$)		Power ($\delta = -0.1$)	
				ADL	*ADL2*	*ADL*	*ADL2*
ΔZ_{t-1}	0	0.9	1	0.015	0.018	0.783	0.842
	3					0.866	0.985
	8					0.982	1.000
	0	0.9	3	0.018	0.017	0.982	0.994
	3					0.975	0.994
	8					0.992	1.000
Z_{t-1}	0	0.9	1	0.014	0.017	0.683	0.740
	3					0.755	0.949
	8					0.990	1.000
	0	0.9	3	0.012	0.017	0.981	0.991
	3					0.966	0.991
	8					0.992	1.000
I(0)	0	0.9	1	0.014	0.015	0.796	0.849
	3					0.877	0.988
	8					0.984	1.000
	0	0.9	3	0.014	0.015	0.987	0.997
	3					0.976	0.995
	8					0.976	0.995
I(1)	0	0.9	1	0.028	0.034	0.820	0.879
	3					0.882	0.990
	8					0.986	1.000
	0	0.9	3	0.027	0.032	0.990	0.998
	3					0.975	0.997
	8					0.992	1.000

Notes: Details of the simulations are described in Section 2.

Table 4: Estimates of the Linear Taylor Rule for Selected Sample Periods

Sample Period	α_0	π_t	y_t	$i_{t\text{-}1}$	$i_{t\text{-}2}$
1970:1 – 2005:4	-0.165	0.213	0.297	0.901	
(Full Sample)	(-0.883)	(4.286)	(6.102)	(27.324)	
1970:1 – 1979:2	-0.527	0.124	0.324	0.985	
(Martin-Burns-Miller)	(-0.854)	(1.013)	(4.392)	(9.443)	
1979:4 – 2005:4	-0.137	0.537	0.350	0.760	
(Volcker-Greenspan)	(-0.750)	(6.297)	(5.169)	(17.669)	
1983:1 – 2005:4	0.010	0.178	0.151	1.285	-0.377
(Volcker-Greenspan: post money base targeting)	(0.083)	(2.756)	(3.760)	(13.905)	(-4.284)
1987:4 – 2005:4	-0.121	0.137	0.209	1.354	-0.396
(Greenspan period)	(-0.998)	(2.383)	(4.223)	(12.643)	(-3.692)

Notes: The *t*-statistics are in parentheses. We eliminated lagged values of $i_{t\text{-}i}$ if they were not significant at the 10% level.

Table 5: Unit Root Tests of the Taylor Rule Variables

Start	**End**		i_t	π_t	y_t
1970:1	2005:4	ρ	-0.055	-0.014	-0.198
		τ_μ	(-1.922)	(-0.990)	(-5.615)
1970:1	1979:2	ρ	-0.560	-0.110	-0.157
		τ_μ	(-3.388)	(-2.558)	(-2.900)
1979:4	2005:4	ρ	-0.044	-0.032	-0.227
		τ_μ	(-1.532)	(-2.311)	(-4.052)
1983:1	2005:4	ρ	-0.031	-0.057	-0.175
		τ_μ	(-1.711)	(-2.345)	(-4.714)
1987:4	2005:4	ρ	-0.042	-0.048	-0.114
		τ_μ	(-2.375)	(-1.519)	(-2.589)

Note: In each case we estimated a model of the general form $\Delta x_t = \alpha_0 + \rho x_{t\text{-}1} + \Sigma\alpha_i\Delta x_{t\text{-}i} + \varepsilon_t$. Lag lengths were chosen using a maximum lag length (i_{max}) of 12. If the *t*-statistic for the last lag was not significant at the 5% level, i_{max} was reduced by one and the equation was re-estimated. τ_μ is the sample value of the *t*-statistic for the null hypothesis $\rho = 0$. With 50 (100) observations, the critical values at the 10% and 5% significance levels are –2.60 (–2.58) and –2.93 (–2.89), respectively. When the tests were conducted without the insignificant values of α_0, the null hypothesis of a unit root could be rejected at the 1% level in every case.

Table 6: Engle-Granger Linear Cointegration Tests

Start	**End**	β_0	β_1	ρ	**lags**
1970:1	2005:4	2.541 (6.622)	1.049 (12.490)	-0.105 (-2.230)	12
1979:4	2005:4	1.442 (4.292)	1.609 (18.002)	-0.149 (-2.652)	12
1983:1	2005:4	0.930 (1.909)	1.811 (10.083)	-0.090 (-2.026)	9
1987:4	2005:4	1.265 (2.320)	1.546 (6.958)	-0.090 (-2.274)	9

Notes: The *t*-statistics are in parentheses. For each sample period, we estimated a potential long-run equilibrium relationship of the form $i_t = \beta_0 + \beta_1\pi_t + e_t$. The second step was to use the estimated residuals to estimate an equation of the form $\Delta e_t = \rho e_{t-1} + \Sigma\alpha_i\Delta e_{t-1} + v_t$. With 50 (100) observations, the critical value at the 10% and 5% significance levels are –3.31 (–3.09) and –3.46 (–3.40), respectively.

Table 7: Johansen Linear Cointegration Tests

Start	**End**	λ_{max}	λ_{trace}	β_0	β_1	α_1	α_2
1970:1	2005:4	9.88	14.35	4.125	0.658	-0.101 (-2.555)	-0.029 (-2.220)
1979:4	2005:4	18.50	22.13	-59.666	29.092	0.003 (2.476)	0.003 (4.215)
1983:1	2005:4	4.90	6.62	2.320	1.161	-0.050 (-2.206)	0.002 (0.113)
1987:4	2005:4	6.88	8.82	4.100	0.151	-0.059 (-2.609)	-0.009 (-0.599)

Notes: We estimated the system $\Delta x_t = \alpha\beta x_{t-1} + \Sigma\Gamma\Delta_i x_{t-i} + \varepsilon_t$ where $x_t = [i_t, \pi_t]'$. We did not include a time trend, constrained the intercepts to appear in the cointegrating relationship, and normalized with respect to i_t. Hence, any cointegrating relationships were assumed to have the form $i_t = \beta_0 + \beta_1\pi_t$. The α_i are the speed of adjustment coefficients (i.e., the factor loadings). The values of λ_{max} and λ_{trace} are usual statistics for the null hypothesis that the number of cointegrating vectors is zero. At the 5% significance level, the asymptotic critical values for λ_{max} and λ_{trace} are 15.67 and 19.96, respectively.

Table 8: Threshold Estimates of the Taylor Rule

	Start End	**Start End**	**Start End**	**Start End**	**Start End**
	1970:1	1970:1	1979:4	1983:1	1987:4
	1979:2	2005:4	2005:4	2005:4	2005:4
α_0	-3.906	-3.209	6.023	1.703	0.001
	(-0.899)	(-1.182)	(3.223)	(1.839)	(0.003)
β_0	-1.097	-0.162	-0.088	-0.048	0.026
	(-3.119)	(-0.986)	(-0.420)	(-0.536)	(0.239)
$I_{1t}\pi_t$	1.928	1.188	0.588	0.931	1.041
	(1.947)	(3.744)	(3.916)	(4.955)	(3.239)
$I_{2t}\pi_t$	0.160	0.149	0.196	0.092	0.053
	(1.291)	(2.872)	(1.692)	(1.578)	(0.925)
$I_{1t}y_t$	1.279	1.300	1.356	0.335	0.638
	(2.471)	(8.911)	(5.266)	(5.504)	(5.148)
$I_{2t}y_t$	0.312	0.231	0.232	0.176	0.209
	(3.912)	(4.921)	(3.607)	(3.939)	(3.696)
$I_{1t}i_{t-1}$	-0.069	0.685	0.458	0.803	0.783
	(-0.200)	(11.594)	(5.309)	(5.197)	(3.724)
$I_{2t}i_{t-1}$	1.038	0.931	0.907	1.455	1.392
	(9.515)	(27.874)	(19.598)	(16.478)	(11.097)
$I_{1t}i_{t-2}$				-0.439	-0.208
				(-2.856)	(-1.235)
$I_{2t}i_{t-2}$				-0.496	-0.442
				(-5.953)	(-3.552)
d	2	1	1	1	2
τ	8.187	8.078	5.517	3.633	2.721
prob	0.144	0.017	0.001	0.000	0.024

Notes: The *t*-statistics are in parentheses.

Table 9: System IV Threshold Cointegration Estimates of the Nonlinear Taylor Rule

	Start/End	**Start/End**	**Start/End**	**Start/End**	**Start/End**
	1970:1	1979:4	1983:1	1987:4	1983:1
	2005:4	2005:4	2005:4	2005:4	2005:4
$\Delta i_t = [\alpha_{11} i_{t-1} + \alpha_{12} \pi_{t-1} + \alpha_{13} y_{t-1}] I_{1t} + [\beta_{11} i_{t-1} + \beta_{12} \pi_{t-1} + \beta_{13} y_{t-1}] I_{2t} \ldots$					
α_{11}	-0.192	-0.632	-0.584	-0.618	-0.564
	(-2.568)	(-5.370)	(-0.532)	(-2.834)	(-3.083)
β_{11}	-0.079	-0.023	-0.074	-0.098	-0.074
	(-0.421)	(-0.245)	(-0.056)	(-2.608)	(-1.245)
α_{12}	0.257	0.793	0.994	1.239	0.676
	(1.372)	(4.725)	(0.317)	(2.406)	(2.243)
β_{12}	0.040	0.031	0.016	0.023	-0.079
	(0.040)	(0.046)	(0.002)	(0.130)	(-0.356)
α_{13}	0.211	0.069	0.256	0.921	0.210
	(1.999)	(0.409)	(0.215)	(3.574)	(2.333)
β_{13}	0.155	0.137	0.196	0.245	0.149
	(0.395)	(1.261)	(0.084)	(3.841)	(2.055)
$\alpha_{11}=\beta_{11}=\alpha_{12}=\beta_{12}=0$	0.095	0.000	0.839	0.002	0.007
$\alpha_{11}=\beta_{11}=0$	0.034	0.000	0.774	0.001	0.004
$\alpha_{12}=\beta_{12}=0$	0.390	0.000	0.793	0.054	0.077
$\Delta \pi_t = [\alpha_{21} i_{t-1} + \alpha_{22} \pi_{t-1} + \alpha_{23} y_{t-1}] I_{1t} + [\beta_{21} i_{t-1} + \beta_{22} \pi_{t-1} + \beta_{23} y_{t-1}] I_{2t} \ldots$					
α_{21}	0.042	-0.032	0.397	0.447	0.251
	(1.226)	(-0.691)	(2.568)	(3.058)	(1.872)
β_{21}	-0.047	0.041	-0.010	-0.013	0.022
	(-0.623)	(1.024)	(-0.187)	(-0.535)	(0.501)
α_{22}	0.014	0.079	-0.083	-1.098	-0.205
	(0.157)	(1.036)	(-0.284)	(-3.108)	(-0.948)
β_{22}	-0.045	-0.078	-0.372	-0.205	-0.031
	(-0.112)	(-0.246)	(-1.466)	(-1.431)	(-0.194)
α_{23}	0.095	-0.004	-0.274	-0.365	-0.146
	(2.175)	(-0.070)	(-3.110)	(-2.067)	(-2.294)
β_{23}	0.077	0.009	0.100	0.052	0.057
	(0.485)	(0.196)	(1.640)	(1.569)	(1.142)
$\alpha_{21}=\beta_{21}=\alpha_{22}=\beta_{22}=0$	0.078	0.624	0.048	0.024	0.404
$\alpha_{21}=\beta_{21}=0$	0.391	0.424	0.033	0.008	0.164
$\alpha_{22}=\beta_{22}=0$	0.981	0.512	0.339	0.004	0.617
$\alpha_{11}=\beta_{11}=\alpha_{22}=\beta_{22}=0$	0.148	0.000	0.615	0.000	0.014
all $\alpha_{ij}=\beta_{ij}=0$	0.019	0.000	0.005	0.000	0.004
Threshold	$\pi_{t-1} = 1.63$	$\pi_{t-1} = 4.32$	$\pi_{t-1} = 3.63$	$\pi_{t-2} = 2.72$	$w\pi_{t-1}+(1-w)y_{t-1} = 2.37$

Notes: The *t*-statistics are in parentheses.

Table 10: Single Equation IV Threshold Cointegration Estimates of the Nonlinear Taylor Rule

	Start	**Start**	**Start**	**Start**	**Start**	**Start**	**Start**	**Start**
	End	**End**	**End**	**End**	**End**	**End**	**End**	**End**
	1970:1	1979:4	1983:1	1987:4	1970:1	1979:4	1983:1	1987:4
	2005:4	2005:4	2005:4	2005:4	2005:4	2005:4	2005:4	2005:4
	without $I_{1t}\Delta\pi_t$ and $I_{2t}\Delta\pi_t$				with $I_{1t}\Delta\pi_t$ and $I_{2t}\Delta\pi_t$			
$I_{1t}i_{t\text{-}1}$	-0.129	-0.630	0.160	-0.181	-0.134	-0.718	-0.302	-0.811
	(-1.97)	(-2.72)	(0.33)	(-1.03)	(-1.77)	(-3.18)	(-2.72)	(-4.98)
$I_{2t}i_{t\text{-}1}$	-0.066	-0.053	-0.029	-0.084	-0.080	-0.062	-0.055	-0.080
	(-1.08)	(-2.26)	(-1.05)	(-2.64)	(-2.34)	(-1.64)	(-1.57)	(-2.36)
$I_{1t}\pi_{t\text{-}1}$	0.247	1.041	-0.042	0.239	0.195	1.032	0.689	1.629
	(1.58)	(3.28)	(-0.09)	(0.63)	(1.53)	(3.43)	(2.21)	(4.13)
$I_{2t}\pi_{t\text{-}1}$	-0.001	0.096	0.080	-0.010	-0.067	0.047	0.050	0.010
	(-0.00)	(0.83)	(0.56)	(-0.07)	(-0.33)	(0.17)	(0.24)	(0.06)
$I_{1t}y_{t\text{-}1}$	0.234	0.389	0.013	0.585	0.189	0.383	0.276	0.982
	(2.80)	(2.67)	(0.04)	(2.67)	(1.82)	(2.76)	(3.85)	(6.24)
$I_{2t}y_{t\text{-}1}$	0.339	0.267	0.315	0.367	0.343	0.289	0.301	0.354
	(5.05)	(7.07)	(5.89)	(5.80)	(5.51)	(6.37)	(4.45)	(5.43)
$I_{1t}\Delta\pi_t$					0.418	0.349	0.840	1.538
					(1.15)	(0.59)	(2.45)	(5.29)
$I_{2t}\Delta\pi_t$					0.243	0.378	0.302	0.173
					(1.20)	(1.85)	(1.58)	(0.86)
α_0	-0.241	1.210	-1.279	0.952	0.148	2.247	-0.217	1.078
	(-0.23)	(0.83)	(-0.57)	(2.08)	(0.14)	(1.24)	(-0.19)	(2.80)
β_0	0.237	-0.042	-0.084	0.233	0.344	0.123	0.044	0.181
	(0.23)	(-0.13)	(-0.25)	(0.64)	(0.73)	(0.17)	(0.09)	(0.45)
$\alpha_{11}=\beta_{11}=0$	0.079	0.006	0.548	0.018	0.014	0.002	0.007	0.000
$\alpha_{11}=\beta_{11}=\alpha_{12}=\beta_{12}=0$	0.019	0.003	0.245	0.011	0.004	0.001	0.003	0.000

Notes: The *t*-statistics are in parentheses.

Appendix. Proof of Theorem 1.
(To be updated)

Suppose that we observe the sample $\{y_t,\ t = 1,..,T\}$, which is a vector of I(1) processes that follow

$$y_t = c_t + x_t,\ \Pi(\mathrm{L})u_t = \varepsilon_t, \tag{A.1}$$

where $\Pi(\mathrm{L}) = \mathrm{I} - \sum_{\mathrm{i=1}}^{\mathrm{p-1}} \Pi_{\mathrm{i}}\mathrm{L}^{\mathrm{i}}$ and $\varepsilon_t \sim iid,\ N(0, \Sigma)$; c_t denotes the deterministic term in the data. We let $y_t = (y_{1t}, y_{2t})'$, where y_{2t} is $k\times 1$. Also, we let

$$u_t = y_{1t} - c_t - \beta' y_{2t}\ , \tag{A.2}$$

where $\hat{u}_t$ is the corresponding residual from the above regression. Then, we consider a sample splitting regression

$$x_t = \theta_1 f_t + v_{1t},\ z_t \geq \gamma \tag{A.3}$$

$$x_t = \theta_2 f_t + v_{2t},\ z_t < \gamma.$$

We define $d_t(\gamma) = \{\ z_t \geq \gamma\ \}$, where $\{\ z_t \geq \gamma\ \}$ is an indicator function such that $d_t(\gamma) = 1$ if $z_t \geq \gamma$ and $d_t(\gamma) = 0$ otherwise. We also define $f_{1t}^* = f_t\, d_t(\gamma)$ and $f_{2t}^* = f_t\, (1\text{-}d_t(\gamma))$. Then, we have

$$x_t = \theta_1 f_{1t}^* + \theta_2 f_{2t}^* + v_t. \tag{A.4}$$

We also let $\theta = (\theta_1', \theta_2')'$ and $f_t^* = (f_{1t}^*, f_{2t}^*)$. Then, the regression for the modified Engle-Granger model (10) can be expressed with

$$a_t = \Delta\hat{u}_t \tag{A.5}$$

$$f_{1t}^* = d_t(\gamma)\ (\hat{u}_{t\,\text{-}1}, \Delta x_t, \Delta z_t) \tag{A.6}$$

$$f_{2t}^* = (1\text{-}d_t(\gamma))\ (\hat{u}_{t\,\text{-}1}, \Delta x_t, \Delta z_t). \tag{A.7}$$

For the regression (7) of the error correction model, we have

$$a_t = \Delta e_t, \tag{A.8}$$

where the expressions for f_{1t}^{*} and f_{2t}^{*} are the same as in (A.6) and (A.7), respectively. The expressions for the autoregressive distributed lagged model (ADL, equation 11) are

$$f_{1t}^{*} = d_t(\gamma)\ (e_{t\,\text{-}1}, x_t, z_t, \Delta x_t, \Delta z_t) \tag{A.9}$$

$$f_{2t}^{*} = (1\text{-}d_t(\gamma))\ (e_{t\,\text{-}1}, x_t, z_t, \Delta x_t, \Delta z_t). \tag{A.10}$$

Therefore, we can use the regression (A.4) for all three different models to test for threshold cointegration with the corresponding expressions for a_t, f_{1t}^{*}, and f_{2t}^{*}. Further, we decompose f_{1t}^{*} and f_{2t}^{*} with the I(1) regressor denoted as $f_t^{(1)}$, and the I(0) regressors denoted as $f_t^{(0)}$ such that

$$f_{1t}^{*} = d_t(\gamma)\ (f_t^{(1)}, f_t^{(0)}) \text{ and } f_{2t}^{*} = (1\text{-}d_t(\gamma))\ (f_t^{(1)}, f_t^{(0)}).$$

For instance, we have $f_t^{(1)} = \hat{u}_{t\,\text{-}1}$ and $f_t^{(0)} = [\Delta x_t, \Delta z_t]$ for the expression (A.6). We define w_t as the instrumental variable for $f_t^{(1)}$. We let $w_t^{*} = (w_{1t}, w_{2t})$, where $w_{1t} = d_t(\gamma)\ w_t$ and $w_{2t} = (1\text{-}d_t(\gamma))\ w_t$.

We assume that ε_t, $t = 1,..,\infty$, is an *iid* process with mean zero, variance σ^2, and finite fourth moment. Define a partial sum process $S_{[rT]} = \Sigma_{\text{j=1}}^{\text{rT}} \varepsilon_j$ with $r \in [0,1]$ and $\xi_t = \varepsilon_{t\text{-}1} + .. + \varepsilon_{t\text{-}m}$, where m is a finite positive integer. Then, we follow Enders, Im, and Lee (2005) to show that

$$T^{\text{-}1}\, \Sigma_{\text{t=1}}^{\text{T}} S_{t\text{-}1}\varepsilon_t \rightarrow 0.5\sigma^2[\text{W}(1)^2 - 1] \tag{A.11}$$

$$T^{\text{-}1/2}\, \Sigma_{\text{t=1}}^{\text{T}} \xi_t\varepsilon_t \rightarrow \sqrt{m}\sigma^2\text{W}(1) \tag{A.12}$$

$$T^{\text{-}1}\, \Sigma_{\text{t=1}}^{\text{T}} \xi_t^{\;2} \rightarrow m\sigma^2. \tag{A.13}$$

The proof is found in the above reference. Letting $\text{F} = \{ f_1^{*}, f_2^{*}, \ldots, f_T^{*}\}$ with $f_t^{*} = (f_{1t}^{*}, f_{2t}^{*})$, we can easily expect that the moment matrix F′F is a diagonal matrix, since $\text{E}(f_{1t}^{*} f_{2t}^{*}) = 0$. Following Enders, Im, and Lee (2005), we define

$$\text{B}_\text{T} = \sum_{\text{t=1}}^{\text{T}} w_t^{*}\, a_t - \sum_{\text{t=1}}^{\text{T}} w_t^{*} f_t^{(0)\prime}[\sum_{\text{t=1}}^{\text{T}} f_t^{(0)\prime} f_t^{(0)}]^{\text{-}1} \sum_{\text{t=1}}^{\text{T}} f_t^{(0)}\, a_t \tag{A.14}$$

$$\text{C}_\text{T} = \sum_{\text{t=1}}^{\text{T}} w_t^{*\,2} - \sum_{\text{t=1}}^{\text{T}} w_t f_t^{(0)\prime}[\sum_{\text{t=1}}^{\text{T}} f_t^{(0)\prime} f_t^{(0)}]^{\text{-}1} \sum_{\text{t=1}}^{\text{T}} f_t^{(0)}\, w_t^{*}. \tag{A.15}$$

Then, using some tedious algebra we can show that

$$\frac{1}{\sqrt{T}} \mathrm{B_T} \rightarrow \sqrt{m}\, \sigma^2 \mathrm{W}(1) \tag{A.16}$$

$$\frac{1}{T} \mathrm{C_T} \rightarrow m\, \sigma^2. \tag{A.17}$$

Therefore, we can show that

$$t_{\rho 1} = \frac{\hat{\rho}_{1iv}}{\mathrm{s}(\hat{\rho}_{1iv})} = \frac{\frac{1}{\sqrt{T}} \mathrm{B_T}}{\hat{\sigma}\sqrt{\frac{1}{T} \mathrm{C_T}}} = \mathrm{W}(1) \sim N(0,1).$$

The distribution for $t_{\rho 2}$ can be obtained in a similar manner. This completes the proof.